Two Models for Parallel Differential Evolution

María Laura Tardivo, Paola Caymes Scutari, Miguel Méndez Garabetti, Germán Bianchini
We present two parallel models for parallelize the **Differential Evolution** (DE) algorithm, based on an island model, a ring interconnection topology and a population migration strategy:

- Subpopulation-based Model
- Island Model

**Goal:** Analyze the performance of both models, with regard to solutions quality versus computing time.

**Test Cases:** The models have been proved with a set of benchmark functions considering different configurations for the parameters of DE.
An optimization problem is defined (Talbi, 2009) by a couple \((S,f)\):
- \(S\) represents the set of possible solutions
- \(f: S \rightarrow R\) is the objective function to optimize.

The **objective function** \(f\) assigns to every solution \(s\) in \(S\) of the search space a real number indicating its worth.

The main goal is to find a solution \(s^*\) in \(S\), called **global optimum**.
Metaheuristics: optimization techniques that provide good solutions to complex problems computed in a reasonable time.

**Differential Evolution** (Rainer Storn & Kenneth Price, 1995)

\[ X_{i,g} = (x_{1,g}^1, ..., x_{N,g}^D) \]
Introduction

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$X_{i,g}$

Mutation

$1 \quad \ldots \quad D$

$X_{best,g}$

$X_{r1,g}$

$X_{r2,g}$

$V_g$
Two Models for Parallel Differential Evolution

Crossover

\[ \text{rand} j \leq C_r ? \]

\[ \begin{array}{cccccccc}
21 & 48 & 95 & 12 & 52 & 32 & 76 & 43 \\
\hline
5 & 13 & 95 & 82 & 52 & 32 & 64 & 43 \\
\end{array} \]

- \( V_g \)
- \( X_{\text{best},g} \)
- \( U_{i,g} \)
- \( X_{r1,g} \)
- \( X_{i,g} \)
- \( X_{r2,g} \)

\[ \text{else} \]

\[ \begin{array}{cccccccc}
5 & 13 & 44 & 82 & 20 & 49 & 64 & 7 \\
\hline
5 & 13 & 44 & 82 & 20 & 49 & 64 & 7 \\
\end{array} \]

- \( V_g \)
Selection

\[ X_{i,g+1} = \begin{cases} U_{i,g} & \text{if } f(U_{i,g}) \leq f(X_{i,g}) \\ X_{i,g} & \text{other case} \end{cases} \]
**Subpopulation-based Model**

**The Models**

- **Master**
- **Subpopulation-based Model**

**Introduction**

**Results**

**Conclusions and future work**

---

Migration of individuals

Return back the best individual found

From node n to node 0

To node 0

The master becomes a worker

**Two Models for Parallel Differential Evolution**

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Conclusions and future work

Island Model

- Initial communication
- Communication to proceed or finalize

Master

- Coordinates tasks
- Does not manage the whole population

node j

node 0

node 1

node n-1

node n

Pop. 0

DE

Pop. 1

DE

Pop. n-1

DE

Pop. n

DE

Migration of individuals

Manages its own population

From node n

To node 0
Suganathan et al, from the CEC’2008 Special Session and Competition on Large Scale Global Optimization.

**Shifted Sphere Function**

\[
F_1(x) = \sum_{i=1}^{D} z_i^2 + f\_bias_1
\]

**Shifted Rosenbrock Function**

\[
F_2(x) = \sum_{i=1}^{D-1} (100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2) + f\_bias_2
\]

Global optimum: \( x^* = 0 \),

\[
F_1(x^*) = f\_bias_1 = -450
\]

\[
F_2(x^*) = f\_bias_2 = 390
\]
The experiments where carried out considering:

- Problems of dimension 100, 500 and 1000.
- Average of 30 executions (mean error).
- Population: 100 and 400 individuals.
- Cr = 0.3, F = 0.5.
- Migration of 15% of the population, every 500 iterations.
- The finalization condition was established to reaching 6000 iterations.
- Ring interconnection topology.
- Scalability measures: 2, 4, 8, 16 and 32 processors dedicated to the workers processes, and a separate processor for the master process for the Island Model.
- The replacement strategy is semi-elitist.
Two Models for Parallel Differential Evolution

### Shifted Sphere
**Dimension 1000**

**Mean error**
- **Sec. 100 ind.:** 9,71e+01
- **Sec. 400 ind.:** 5,85e+03
- **1,00e-03**
- **1,22e+04**

**Mean time**
- **1,00e-03**
- **1,22e+04**

**Numeric results**

**Processors**
- 1, 2, 4, 8, 16, 32

**Mean error**
- **0.001**
- **0.00585**
- **0.0122**

**Mean time**
- **0.001**
- **0.0122**

**Shifted Sphere**
- **Dimension 1000**
- **Subpop. 400 ind.**
- **Island 100 ind.**
- **Island 400 ind.**
- **Sec. 100 ind.**
- **Sec. 400 ind.**

**Bigger magnitude**
- **Island 400 ind.**
- **Island 100 ind.**
- **Subpop. 400 ind.**
- **Sec. 100 ind.**
- **Sec. 400 ind.**
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Two Models for Parallel Differential Evolution

Numeric results

**Shifted Rosenbrock**

*Dimension 1000*

Mean error

Mean time

Bigger magnitude

- Island 400 ind.
- Island 100 ind.
- Subpop. 400 ind.
- Sec. 100 ind.
- Sec. 400 ind.

Processors

Mean error

2,30e+09

9,03e+08

3,15e+06

Processors

Mean time

250.00

150.00

100.00

50.00

0.00

1 2 4 8 16 32

1 2 4 8 16 32
**Shifted Sphere**

**Dimension 500**

**Numeric results**

<table>
<thead>
<tr>
<th>Processors</th>
<th>Mean error</th>
<th>Mean time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.56e-10</td>
<td>5.24e+01</td>
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<td>2</td>
<td>1.39e-03</td>
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<td>16</td>
<td>1.52e+01</td>
<td>1.52e+01</td>
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<tr>
<td>32</td>
<td>7.60e+01</td>
<td>7.60e+01</td>
</tr>
</tbody>
</table>

**Bigger magnitude**

- Island 400 ind.
- Island 100 ind.
- Subpop. 400 ind.
- Sec. 100 ind.
- Sec. 400 ind.
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Two Models for Parallel Differential Evolution

Numeric results

**Shifted Rosenbrock**
**Dimension 500**

**Mean error**

**Mean time**
**Shifted Sphere**
*Dimension 100*

**Numeric results**

<table>
<thead>
<tr>
<th>Processors</th>
<th>Mean error</th>
<th>Mean time</th>
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<td>2.78e-17</td>
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<tr>
<td>32</td>
<td>1,89e-14</td>
<td>10.00</td>
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</tbody>
</table>

**Bigger magnitude**

- Island 400 ind.
- Island 100 ind.
- Subpop. 400 ind.
- Sec. 100 ind.
- Sec. 400 ind.
**Shifted Rosenbrock**

**Dimension 100**

**Mean error**

<table>
<thead>
<tr>
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<th>Sec. 100</th>
<th>Sec. 400</th>
<th>Is. 100 ind.</th>
<th>Is. 400 ind.</th>
<th>Subpop. 400 ind.</th>
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</thead>
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<td>1</td>
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<td>2</td>
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**Mean time**

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<th>Island 400 ind.</th>
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<th>Sec. 100 ind.</th>
<th>Sec. 400 ind.</th>
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</tbody>
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Bigger magnitude

- Island 400 ind.
- Island 100 ind.
- Subpop. 400 ind.
- Sec. 100 ind.
- Sec. 400 ind.
Through the results analysis it was found that:

• The subpopulation model reduces significantly the computing time, but the quality of the solutions is not the optimal that can be achieved.

• With the island model, the computing time is not reduced, because of the model characteristics, but the solutions quality is improved significantly. This feature reflects the fact that the model explores a greater search space, since each island is configured with a different initial seed.

As future works we plan to employ these parallelization models to replace the sequential DE scheme in a hybrid metaheuristic that combines DE with Local Search.
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